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THE KINETICS OF THE SILO'S WALLS DURING THE FLOW OF A BULK MATERIAL

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ABSTRACT

Both static pressure and dynamic pressure act on the silo's walls. The static pressure is usually regular and can be determined on the basis of the material specifications and the relevant test results. The dynamic pressure can be more complex. It can have the form of pulse loads generated by the collapse of stored material archings, a nonuniform horizontal pressure stemming from the physical properties of the material, and variable pressures produced by, i.a., the energy generated by the friction of the flowing material against the silo walls. The latter factor induces vibrations which can be classified as self-excited vibrations (self-oscillations). This paper presents a mathematical model for describing the displacement of the silo walls dynamically loaded with a material flowing in the silo as it is emptied. The considerations are based on the results of the research conducted for several decades by Professor Augustyn Borcz's team at Wrocław University of Science and Technology

KEYWORDS: silo, wall kinematics, self-excited vibrations.

1. INTRODUCTION

Calculations of the pressure exerted on the silo's walls play a major role in the design of (prestressed) reinforced concrete silos, steel silos and silos made of composite materials. The conclusions emerging from the research on the pressure exerted on the silo walls by a loose bulk material conducted by Professor Augustyn Borcz's team in the 1970s and 80s are not distorted by the scale effect since the investigations were carried out on existing silos. Owing to the large number of tests carried out in the course of material storage and flow, the parameters essential for determining the pressure exerted on the walls of silos were identified[1], [2], [3], [4].

As a result of the above research Prof. A. Borcz verified the assumptions (still used today) of Janssen's theory, based on the axial symmetry of the silo load generated by vertical pressure p_v and the uniform distribution of horizontal pressure p_h which is proportional to the vertical pressure.

$$p_h = kp_V \quad (1)$$

In the above equation k is a proportionality factor independent of the silo filling level. A constant value (the same as at rest) is also assumed for the coefficient of friction (μ_1) of the stored material against the silo wall. Prof. A. Borcz found Janssen's assumptions to be an oversimplification and that the results of the calculations were not always close to the values measured in the full-scale silo. In his investigations, conducted for over 30 years, he paid attention to both the accuracy of the measuring methods and the sensitivity of the parameters used to analytically determine the values of the pressure. In his laboratory studies he also paid attention to the geometric similarity to the full-scale silo, and to the model scale.

Most of the pressure measurements were performed on full-scale silos storing organic and inorganic bulk materials. Sometimes it is the kind of material which determines the choice of parameters for calculating the pressure exerted on the silo walls. For example, when calculating the pressure exerted by grain seeds one can observe a change in bulk density depending on the pressure exerted by the layers situated above. In the case of cement storing, one should take into account the thermal effects associated with the storage of water absorbing

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materials. Additional pressures, which are difficult to describe explicitly, occur during the flow of the material through the silo. Weight-relieving and aiding devices play an important role here.

2. PHYSICAL PROPERTIES OF MATERIALS STORED IN SILO

On the basis of the literature on the subject the following were recognized as the most important parameters (but not the only ones) determining the horizontal pressure exerted on the silo walls:

- the density / the specific gravity,
- the internal friction angle,
- the coefficient of friction of the material against the wall.

2.1. Bulk density and specific gravity

Bulk density does not describe a material unambiguously since this quantity depends on, among other things, the degree of compaction of the material and the humidity and temperature at which it is stored. Also the duration of storage is important since the drying of the material, the vibrations of the foundation caused by external factors and the biological processes taking place in organic materials contribute to a change in bulk density. For these reasons specific gravity would describe the material more explicitly, but it is difficult to determine for many organic materials and therefore for technical purposes the following much simpler formula for bulk density is most often used:

$$\gamma_0 = \frac{G}{V} \tag{2}$$

□$_0$–bulk density [N/m^3],
G –the weight of the sample [N],
V –the volume of the sample [m^3].

Because of material inhomogeneity, it is necessary to introduce at least some simple statistical quantities, e.g. the mean value and the mean standard deviation, and to determine the confidence level.

$$\gamma_0 = \bar{\gamma}_0 \pm t\bar{s}_{\gamma_0} \tag{3}$$

$\bar{s}_{\gamma_0}$ - the mean density,
$\bar{\gamma}_0$ - the mean standard deviation,
t - the confidence level.

2.2. Internal friction angle

In order to determine the shear strength of the stored material one needs to determine the internal friction angle and the cohesion. Mohr's circle shows the mutual relations between normal stresses and shear stresses (fig. 2).

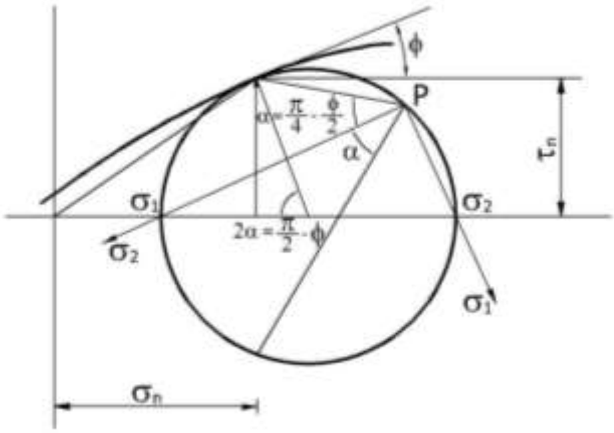

Fig. 2. Mohr's circle –normal stresses, shear stresses and internal friction angle.

Shear strength can be expressed by the Coulomb formula:

$$t = \tau = \sigma_n \cdot \text{tg}\ \varphi + c \qquad (4)$$

where:

- t – the shear strength [Pa],
- τ–the shear stress [Pa],
- σn –the stress normal to the shear surface [Pa],
- tg φ–the internal friction coefficient,
- φ–the internal friction angle [rad],
- c –the cohesion [Pa].

2.3.Coefficient of friction of bulk material against silo wall

The coefficient of friction of a bulk material against the silo wall is a ratio of the shearing force inducing slip of the two materials relative to the normal force pressing the bulk material against the wall:

$$\mu_1 = \frac{T}{N} \qquad (5)$$

where:

- μ_1 –the coefficient of friction of the material against the wall [-],
- T – the shearing force inducing slip [N],
- N – the force normal to the wall surface [N].

The auxiliary parameters used to describe materials stored in silos are: porosity, compressibility and moisture content.

3. CLASSICAL METHOD OF CALCULATING PRESSURE EXERTED BY BULK MATERIAL

A static pressure is exerted on the silo walls and bottom when the bulk material is in a static state, i.e. in the storage phase (at rest). The scheme of equilibrium conditions in the silo is based on the differential equation formulated by Janssen, i.a.,[1], [5], [7]:

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$$\frac{dp_v}{dz} + p_v\, k\, tg\varphi' \frac{U}{F} = \gamma$$

where:

- p_v – the vertical pressure,
- z – the fill level elevation,
- φ'–the angle of friction of the material against the wall,
- U – the circumference of the chamber,
- F – the cross sectional area of the chamber,
- γ – the bulk density of the bulk material.

By solving the above differential equation Janssen obtained formulas for the vertical pressure and the horizontal pressure:

$$p_v = \frac{\gamma F}{k\, tg\varphi' U}\left(1 - e^{-\frac{k\, tg\varphi' U z}{F}}\right) \tag{11}$$

and

$$p_h = \frac{\gamma F}{tg\varphi' U}\left(1 - e^{-\frac{k\, tg\varphi' U z}{F}}\right) \tag{12}$$

Since Janssen assumed the horizontal/vertical pressure ratio to be constant, whereas the results of many tests showed this ratio to be variable, Janssen's theory is used with Koenen's correction [8], which takes into account this variability as follows:

$$k = tg^2\left(45^\circ - \frac{\varphi}{2}\right) \tag{13}$$

φ – the internal friction angle for the stored material.

The static pressure according to M. and A. Reimbert is based on the same assumptions as the Janssen method, with the difference that the exponential function in Janssen's formula has been replaced with a hyperbolic pressure function approximating pressure values on the basis of experimental results. For cylindrical chambers the formulas for vertical pressure and horizontal pressure are as follows:

$$p_v = \frac{r_h}{k\, tg\phi'}\left[1 - \left(\frac{z}{A} + 1\right)^{-2}\right] \tag{14}$$

$$p_h = \frac{r_h}{tg\phi'}\left[1 - \left(\frac{z}{A} + 1\right)^{-2}\right] \tag{15}$$

$r_h = \frac{F}{U}$ – the hydraulic radius,

$$A = \frac{r_h}{tg\varphi' k} - \frac{h}{3}$$

h– the height of the upper cone of the bulk material.

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4. DYNAMIC PRESSURE OF BULK MATERIAL EXERTED ON SILO WALLS

Measurements of the pressure exerted on the silo walls show that besides the static pressure, which decreases as the material flows out of the silo, there occurs more or less regular oscillating motion around the line defined by the static pressure. The dynamic pressure exerted on the silo walls and bottom occurs when the bulk material is in motion, particularly as the silo is being emptied. There are various hypotheses on which the models describing the phenomena connected with the increased pressure exerted on the silo walls are based [5], including:

– the hypothesis of dynamic archings – adopted by, among others, Nanninga [9], Platonov [10] and Peschl [11],
– the hypothesis of dynamic slides – [12],
– the hypothesis based on the division of the loose bulk medium by vertical planes into areas moving at different rates in the silo chamber [13].

The phenomena occurring during the emptying of silos, particularly the dynamic pressure, have been investigated also by Borcz, Kmita, Maj, Suleja and Ubysz [2, 7, 9], who on the basis of tests have observed that:

– the increased dynamic pressure is distributed within the hopper area, and particularly in the chamber's bottom part;
– the dynamic pressure can assume values as high as the double static pressure value and it can be even four times higher than the analytically calculated pressure;
– the dynamic impact which occurs in the dumping hopper region has a different character at the beginning of emptying than in the course of this process when the medium is already in motion (is flowing out of the silo).

The finding concerning self-induced vibrations, reported in [1, 2], is of key importance for the present research. The increased pressures resulting from the vibration of the walls appear at the critical rate of flow of a bulk material out of the container. This effect is observed in mainly silo walls and to a lesser degree also in hoppers (fig. 4). According to the authors of [1, 2], this critical rate should be determined as a function of the bulk medium's parameters and the stiffness of the silo walls, and a proper silo design which would rule out the possibility of self-induced vibration generation should be developed.

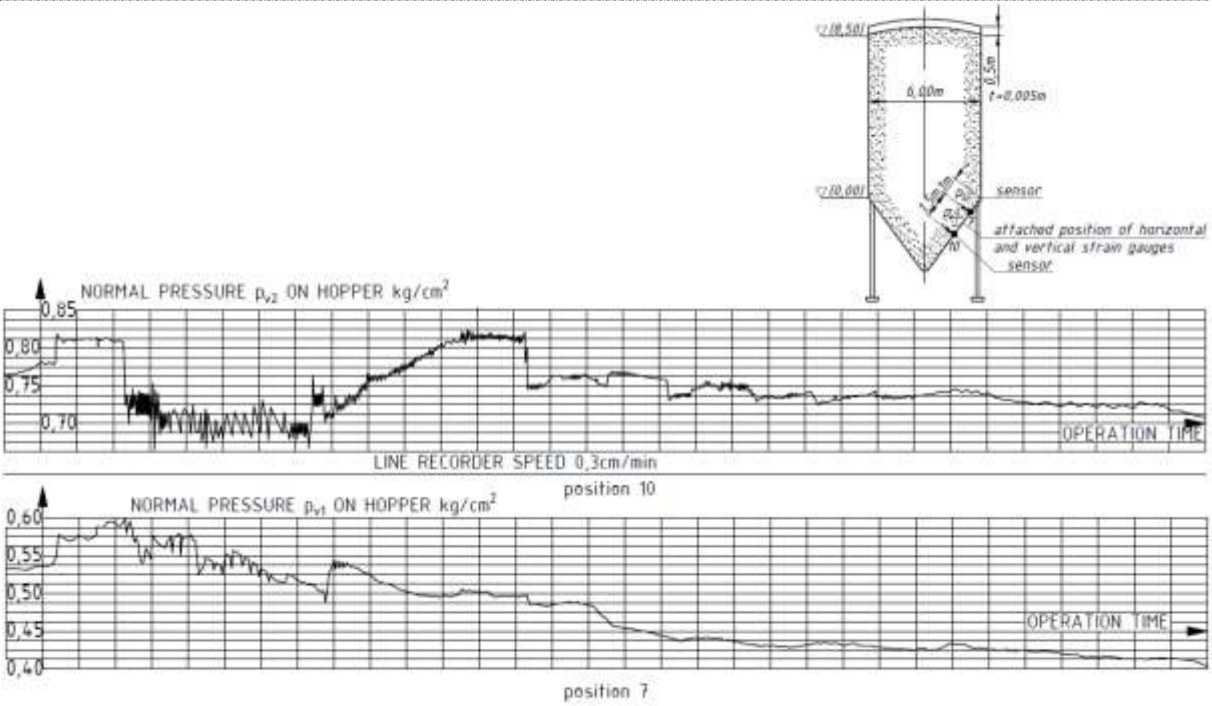

Fig. 4. Circumferential and meridional hopper strains [1].

One should note that the pressures registered during emptying have a distinctly pulsatory character. As the silo is being emptied, the material flowing down at a uniform rate rubs against the silo walls, acting on themdynamically whereby they vibrate. As a result of the mutual interactions between the medium and the walls, the pressure exerted by the medium on the silo walls becomes pulsatory.

5. MODELS DESCRIBING VIBRATION OF SILO WALLS DURING MATERIAL FLOW

5.1. General description of the phenomenon

The pressure exerted on the silo walls has a complex character. One can distinguish:

- static loads,
- pulse loads,
- quasi-harmonic loads,
- loads having a variable random character.

Computational models are usually applied to phenomena exhibiting some regularity. Therefore the description of the considered phenomenon focuses on static loads and quasi-harmonic dynamic loads. Special attention was devoted to the formulation of a computational model describing one kind of the dynamic loads occurring during the flow of a material through the silo, which can be classified as self-excited vibrations.

A self-excited system is a nonconservative system which can perform nondecaying periodic motion. The important and common cases of self-oscillations are connected with the occurrence of dry friction. The amplitude and frequency of self-excited vibrations are determined by the system parameters alone and do not depend on the initial conditions. The most characteristic property of a self-excited system is that the energy lost due to friction is replenished (fig.5).

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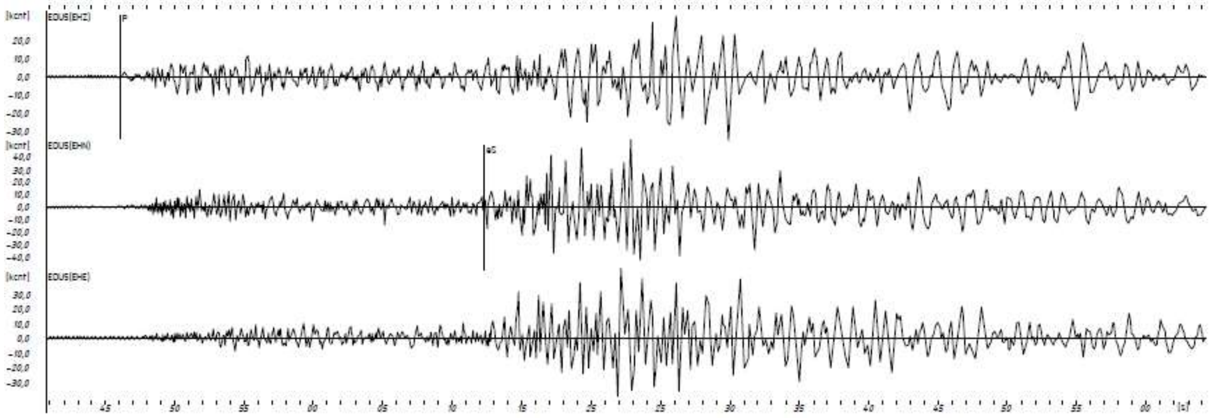

Fig. 5. Exemplary traces of increasing and decreasing self-excited vibrations.

5.2. Description of self-excited vibration in silos

When formulating a relevant model two characteristic cases in which cyclically excited pressure is exerted on the silo walls were distinguished. In the first case, the phenomenon occurs in a silo with deformable walls while the medium can be hardly deformable (e.g. sand). In the second case, the material is deformable (e.g. sugar beet seeds) while the silo walls can be stiff. In both cases, the pressure exerted on the silo walls is a function of time and can be presented as the superposition of:

a) a laminar function representing the mean pressure exerted on the walls during the flow of the material out of the silo and
b) a function representing pulsing pressures exerted on the silo walls, relative to the laminar function.

Depending on the resonance frequencies of the silo and the velocity of the bulk material, the dynamic effects stemming from self-oscillations can exceed other effects and even cause damage to the silo.

So far the self-oscillation phenomenon has not been described in the form of a mathematical computational model which would enable its quantitative analysis. For the present author the starting point were the equations of motion for one-dimensional continuous systems together with the assumptions made when deriving these equations in dynamics. The reason for such a model is the silo geometry for which in most cases high polarization in strain and stress distributions is observed.

A characteristic assumption made to describe self-excited vibrations is that the displacements perpendicular to the silo wall are proportional to an appropriate value of the pressure dependent on the horizontal wall displacements and their derivatives (the velocity and acceleration of the vibrations).An important part of the analysis is the formulation of the right side of the equation. By introducing a large number of parameters the phenomenon can be analysed more thoroughly and more precisely described. However, in many cases a large number of parameters makes it impossible to obtain a closed solution, which was the present author’s goal in this regard. Solutions for the following two cases are presented below:

a) the wall is flexible and its weight is negligibly small relative to the stored material – a membrane model applicable to, e.g., grain silos made of sheet metal;
b) the wall is solid and its stiffness should be taken into account under bending – a model applicable to, e.g., reinforced concrete silos in which bulk materials (cement, sugar) are stored.

5.3. Linear membrane model of self-excited system

A tendon fixed at its two opposite ends was adopted as an elementary model for calculations. The tendon is a simplified model of a silo wall cylindrical in cross section and characterized by negligibly small flexural stiffness. It is the simplest model which makes it possible to illustrate the idea of self-excited vibrations. Figure 6 shows a wall section acted on by the horizontal static pressure of a bulk material (or grain) ($p_{h1}(x,t)$)and by quasi-harmonic pressure$p_{h2}(w,x,t)$during the flow of the bulk material. Moreover, N(x,t), N(x+□x,t) express a local longitudinal force while□(x,t), □(x+□x,t) represent the deviation from the plumb. Membrane load $p_h(x,t)$ is a transverse force per unit length. From equilibrium condition□x (the vertical axis), which takes into account the forces acting during motion and the inertial forces (d’Alembert’s principle), one gets:

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$$N(x+\Delta x,t) \sin \alpha (x+\Delta x,t) - N(x,t) \sin \alpha (x,t) = \Delta x \left[m \frac{d^2 w(x,t)}{dt^2} - p_h (w,x,t)\right] \quad (16)$$

where:

m – the wall weight per unit length,
w – the displacement in the direction perpendicular to the axis,
$\frac{d^2 w(x,t)}{dt^2}$ – the acceleration,
$p_h (x,t) = p_{h1} (x,t) + p_{h2} (w,x,t)$,
$p_{h1}(x,t)$ – the horizontal static pressure of a bulk material (or grain),
$p_{h2}(w,x,t)$ – the quasi-harmonic pressure during the flow of the bulk material.

Expression $m \frac{d^2 w(x,t)}{dt^2}$ represents the inertial force stemming from Newton's second law.

Dividing both sides of equation 16 by Δx one gets a differential quotient on the left side. Assuming $\Delta x \rightarrow 0$ and that for small angles *in* sin $\alpha \rightarrow dw/dx$ one gets the equation:

$$Nw''(x,t) - m \frac{d^2 w(x,t)}{dt^2} = - p_h(w,x,t) \quad (17)$$

However, the above equation has a rather general character. In order for the excitation force to induce self-oscillations it must depend on the wall displacement or its derivatives. In the considered case, it must depend on the wall displacement consistent with the direction of the material pressure (the direction perpendicular to the wall surface)and on the rate of flow of the material in the silo. Some authors (e.g. Ryczek [15]) assume static friction force F_s as an auxiliary parameter. But one should clearly separate the action of the forces occurring in the self-excited system from the static or quasi-static pressure occurring during storage and silo filling and emptying. Hence the right side of the differential equation is presented as the sum of loads $p_{h1}(x,t)$ and $p_{h2}(w,x,t)$, in which the second summand (term) describes the self-excited vibrations:

$$Nw''(x,t) - m \frac{d^2 w(x,t)}{dt^2} = - p_{h1}(w,t) - p_{h2}\left(w, \frac{dw}{dt}, \frac{d^2 w}{dt^2}, x, v_{bm}, F_s, t\right) \quad (18)$$

v_{bm} – the velocity of the loose bulk material flow at the wall,
F_s – the static force of the friction of the bulk material along the silo wall.

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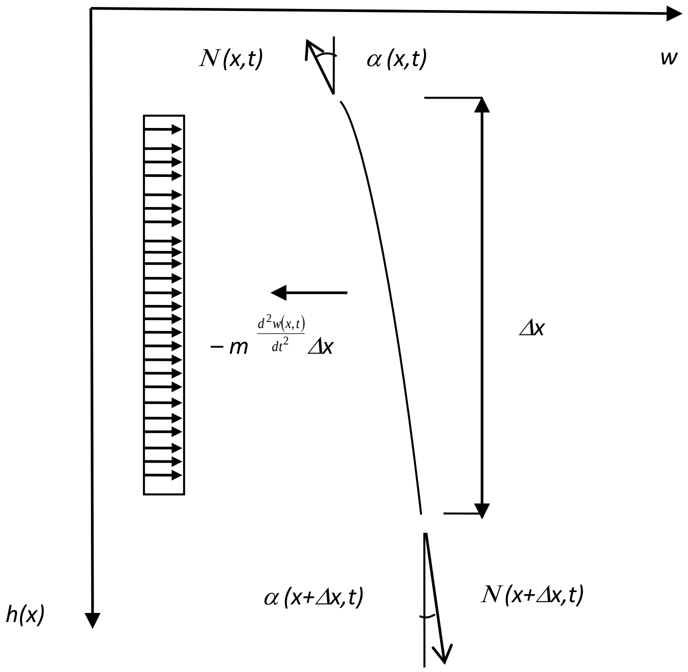

Fig. 6. Membrane segment and distribution of forces.

5.4. Linear flexural model of self-excited system

In the case of a silo with an inflexible wall, one should also take into account its stiffness. Similarly as for a slender wall, a wall band with unit stiffness is assumed. From the general materials strength relations and the geometrical relations and by taking into account the physical conditions one gets for small displacements:

$$M(x,t) = -EI(x)\,\frac{d^2 w(x,t)}{dx^2} \tag{19}$$

Similarly as in the case of the silo with a slender wall and using similar algebraic transformations, one gets the following equation of motion for an elastic beam:

$$EI(x)\left[\frac{d^4 w(x,t)}{dx^4}\right] + m\,\frac{d^2 w(x,t)}{dt^2} = p_h(w,x,t) \tag{20}$$

After the load being the sum of the laminar function and the oscillatory function, describing the process of self-excited vibrations, is taken into account, the equation assumes this final form:

$$EI(x)\left[\frac{d^4 w(x,t)}{dx^4}\right] + m\,\frac{d^2 w(x,t)}{dt^2} = p_{h1}(x,t) - p_{h2}\left(w, \frac{dw}{dt}, \frac{d^2 w}{dt^2}, x, v_{bm}, F_s, t\right) \tag{21}$$

6. SOLUTIONS OF EQUATIONS DESCRIBING SILO WALL VIBRATIONS

The self-excited system models were based on the equation of motion. A solution, which would describe the character of the pressures exerted on the silo wall, was sought in a closed form. The silo wall displacement function was found to depend on the time-dependent pressure values.

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6.1. Wall displacements for laminar flow

The displacements of the walls of a silo when no pressure oscillations occur in it (the material flow through the silo is laminar) can be described by an exponential equation. The equation depends on the level at which the pressure is measured and on the height of the stored material. Because of the complexity of the factors influencing the actual pressure values, conventional normalized pressure $p_h = 1$ was assumed. For a particular silo this pressure should be multiplied by the value of the actual pressure at the given level, determined on the basis of, e.g., Janssen's assumptions. The figures below show exemplary graphs of the displacements of the silo wall in the direction perpendicular to its surface versus time since the start of the outflow. In order to better illustrate the functions, the changes in the displacements are shown for three levels: in respectively the top, middle and bottom part of the silo. The function indicates, among other things, an increased pressure as the flow begins. The pressure is the highest in the silo's bottom part.

Because of the large number of factors having a bearing on the pressure values at the particular levels of the silo, the following function describing the wall displacements in the parametric form is to be used:

$$w(x,t) = mt^L e^{\frac{-ht}{n\pi}} + c_1 t + c_2 \tag{22}$$

For the initial conditions:

$$w(x,t_0) = 1; \quad w(x,t_e) = 0; \quad \frac{\partial w(x,t_0)}{\partial t} = 0; \quad \frac{\partial w(x,t_e)}{\partial t} = 0$$

t_0 – the start of silo emptying,
t_e – the end of silo emptying,

one gets a relation describing the changes in silo wall displacements over time. The initial and boundary conditions for calculating the parameters and the integration constants are determined on the basis of experimental results. For further description the following values of the parameters with low sensitivity to the course of the function are assumed:

m –the coefficient describing the initial phase of the flow<0.7÷1.00> – amounting to 0.8,
L –the coefficient describing the shape of the function – amounting to 0.1,

$$H = \frac{h}{n\pi},$$

h –the coefficient describing the shape (convexity, concavity, point of inflexion) of the function – amounting to 0.005.

It was assumed that the material starts to flow out at instant t_0. Then the function was assumed to have the following form:

$$w(x,t) = \begin{cases} 1 & t \le t_0 \\ mt^{0,1} e^{\frac{-0,005t}{n\pi}} + c_1 t + c_2 & t > t_0 \end{cases} \tag{23}$$

The shape of the function and the parameters were based on experimental data. This solution allows one to change the parameters for the flows of other materials.

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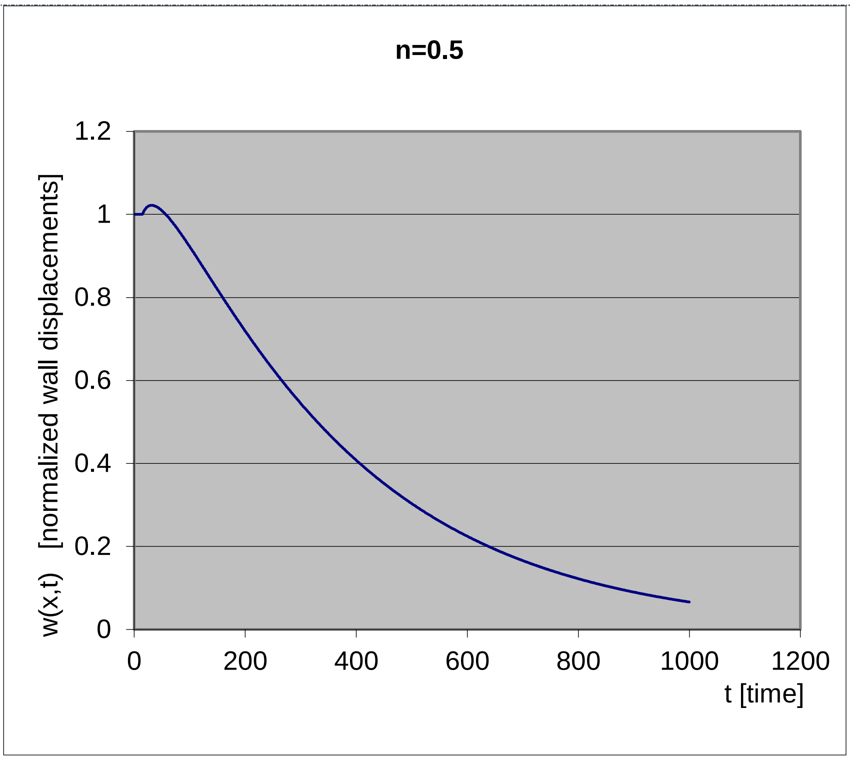
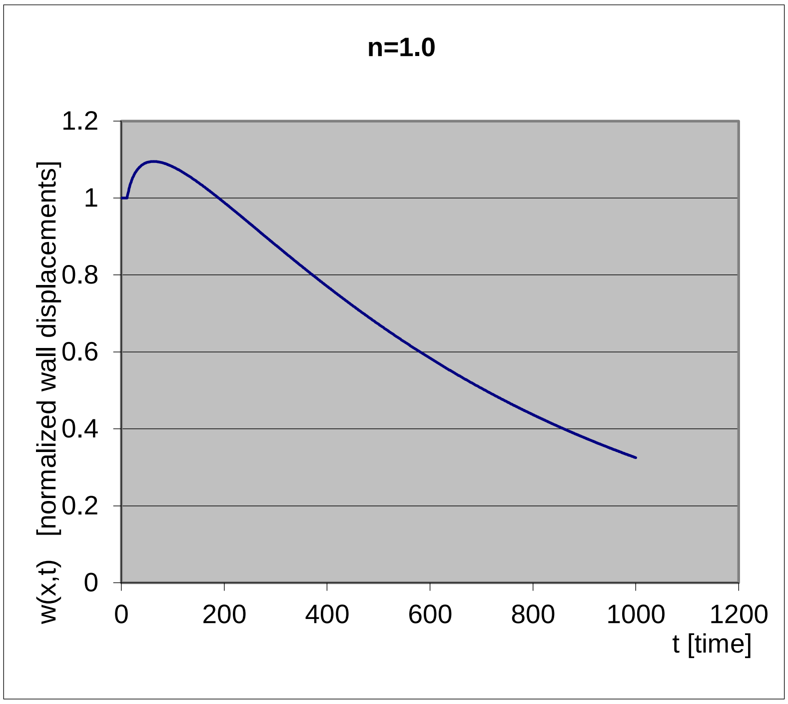


$$w_1(x,t) = 0.8t^{0.1}e^{\frac{-0.005t}{0.5\pi}}$$

Fig.7. Horizontal wall displacements versus time (top silo segmentn=½).



$$w_1(x,t) = 0.8t^{0.1}e^{\frac{-0.005t}{\pi}}$$

Fig.8.Vertical wall displacements versus time (middle silo segmentn=1).

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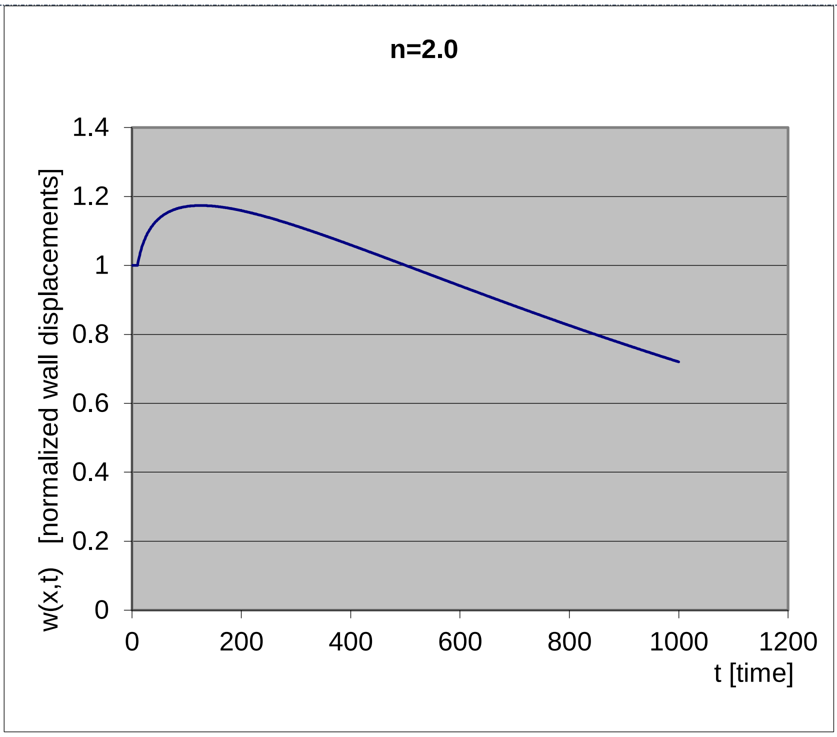

$$w_1(x,t) = 0.8t^{0.1}e^{\frac{-0.005t}{2\pi}}$$

Fig.9.Horizontal wall displacements versus time (bottom silo segment n=2).

6.2. Wall displacements for flow with self- excited vibrations

The displacements of the wall of a silo in which oscillations occur can be described by a differential equation having the form of a general equation of motion. In the general form of the equation the excitation force is assumed to be proportional to the silo wall's horizontal displacements, velocities and accelerations.

$$\frac{\partial^2 w(x,t)}{\partial t^2} + t^n \frac{\partial w(x,t)}{\partial t} + w(x,t) = \\ = p_{h1}(x,t) + p_{h2}\left(w, \frac{\partial w}{\partial t}, \frac{\partial^2 w}{\partial t^2}, x, v_{bm}, F_s, t\right) \quad (24)$$

where: n X<½; 2> (an experimentally determined parameter).

The initial conditions were assumed as for the laminar flow. Assuming then that the displacements of the wall are proportional the pressure exerted on this wall, one gets the dependence between the displacements and the pressure and the time of the flow of the material out of the silo. Depending of the assumed value of*n*(in most cases within the interval<½; 2>), the pressure values will decay at different times.

The effect of the vibrations on the pressure values depends on many factors and can vary both quantitatively and qualitatively. Some of the factors are:

- the material's cohesiveness (its moisture content, degree of granulation, degree of agglomeration),
- the angle of internal friction of the bulk material,
- the coefficient of friction of the material against the wall,
- the flexibility of the silo walls,
- the boundary conditions (the way in which the walls are joined together, the way in which the silo rests on the supports, the foundation conditions),

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- the velocity of the flow of the material,
- the disturbances of the flow (relieving devices, material inhomogeneity).

The flow of the material out of the silo is often disturbed by the dynamic factors accompanying this process, such as:

- collapses caused by the caving in of archings,
- vibrations accompanying flow turbulence.

–

The analytical model applies to idealized vibrations induced by dry friction, also known as self-excited vibrations.

The equation describing the vibration of the silo wall over time was solved using the prediction method. A function being a superposition of the mean laminar pressure and the wave describing the oscillations was plotted. Besides the conditions enabling frequency and amplitude modulation, also conditions enabling the description of vibration excitation and damping were superimposed on the function describing the oscillatory wave. The form of the function describing the horizontal displacements of the wall at a selected level is expressed by the equations:

$$w(x,t) = w_1(x,t) + w_2(x,t) \tag{25}$$

where:

$$w_1(x,t) = Mt^L e^{\frac{-ht}{n\pi}}$$

$$w_2(x,t) = Ate^{-Bt}(\sin Ct - \sin Dt)$$

Hence the solution has the form:

$$w(x,t) = Mt^L e^{\frac{-ht}{n\pi}} + Ate^{-Bt}(\sin Ct - \sin Dt) \tag{26}$$

The above function is the particular integral of equation (15). The equation parameters and the integration constants are determined on the basis of experimental results. The parameters in the equations are physically interpreted as follows:

- A – the vibration amplitude; in the description of the pressures exerted on the silo wall, where the amplitude of the oscillatory part of the pressure to the constant part amounts to 5 ÷ 20 %, the values of parameter A are in the range <0.0005 ÷ 0.002>;
- B=b/n□; b – the damping range; n = <½; 2>; in the case when the self-excited vibrations cease at 0.7 of overall pressure p_h exerted on the silo wall, parameter b = 0.03 is assumed; if the self-excited vibrations cease at 0.2 of overall pressure p_h, b = 0.07 is assumed;
- C, D – the parameters (being a function of the velocity of the flowing material) responsible for the resonance excitation frequency; the values closest to the actual excitations are obtained at C/D = <0.85 ÷ 0.95>;

and the ones previously assumed for the laminar flow

m = 0.8;

L = 0.1;

$$H = \frac{h}{n\pi}$$

h = 0.005.

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The pressure was assumed to be proportional to the horizontal displacements of the silo wall, but if self-excited vibrations occur, also an oscillatory load occurs (in addition to the primary pressure occurring as the material flows out of the silo). Also the dependence between the pressure and the time of the flow of the material out of the silo is derived under this assumption. In order to make the comparison of the function graphs possible, the same range of variation was assumed for parameter n, i.e. n = <½; 2>. Thus the pressures can be described at different silo wall levels. Also these models assume conventional normalized pressure ph = 1, which for a particular silo should be multiplied by the actual pressure value at the given level. The next figures (figs 10, 11, 12) show exemplary horizontal wall displacement-time diagrams.

The function describing the displacements of the silo wall is described as the superposition of the carrier wave and the displacements caused by the oscillatory pressures. The introduced parameters enable one to experimentally determine the shape of the function's graph and its expected values. The material begins to flow out at instant t_0. The function is assumed to have the following form:

$$w(x,t) = \begin{cases} 1 & t \le t_0 \\ Mt^{0,1} e^{\frac{-0,005t}{n\pi}} + Ate^{-Bt}(\sin 0{,}9t - \sin 1{,}0t) & t > t_0 \end{cases} \qquad (27)$$

For rigid silos characterized by a high damping decrement, the vibrations have a character of a short-duration excitation resulting in an instantaneous overloading of the silo wall. In this case, the possible overloading of the silo above the level of 1.2 p_h was modelled. Depending the wall flexibility, the description of the vibrations in the initial phase of material flow looks like the one shown in figure 10, with a smaller or larger amplitude. After the initial excitation practically no changes caused by cyclic excitations are registered. In the case of less stiff walls, but with a large damping decrement, the vibrations in the initial stage of the outflow have a higher amplitude, but they quickly decay.

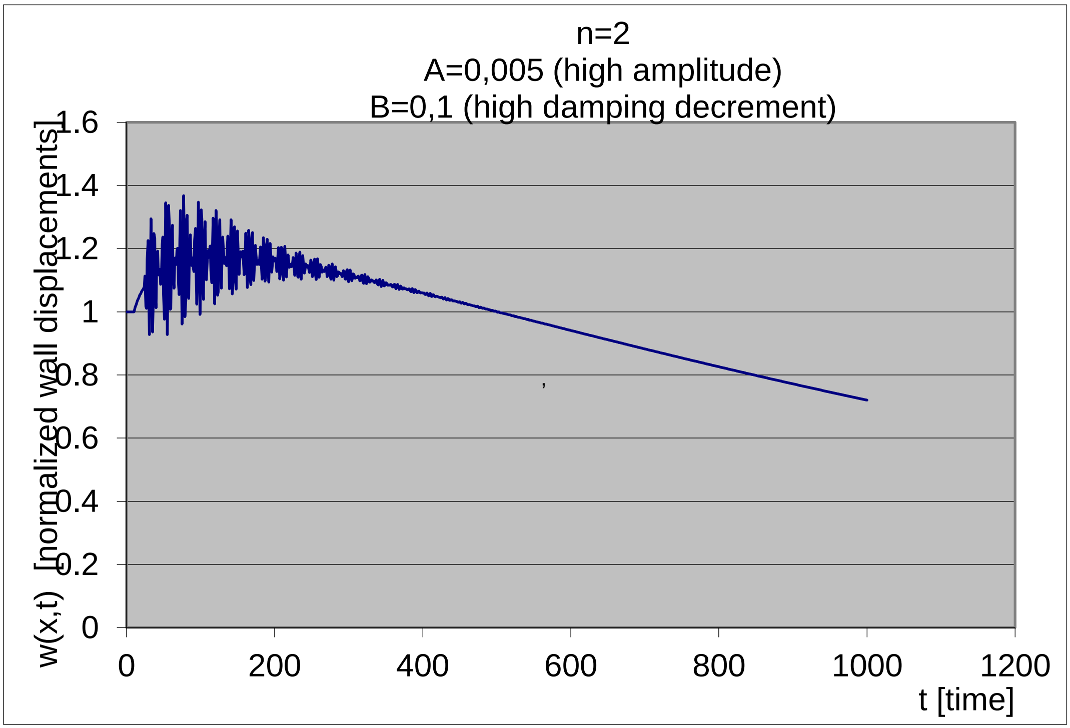

Fig.10. Horizontal wall displacements at high damping decrement (self-excited vibrations can be observed at beginning of outflow).

Another example is a dynamically compliant wall with a low damping decrement, in which cyclic vibrations with a relatively high amplitude are periodically induced. This description of vibrations is quite characteristic of

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self-excited vibrations (fig.11).At a different rate of outflow, self-oscillations can arise over a longer distance. The distance can be modelled using parameters C and D (fig. 12). The modelling of the parameters largely depends on the frequency of the free vibrations of the silo walls.

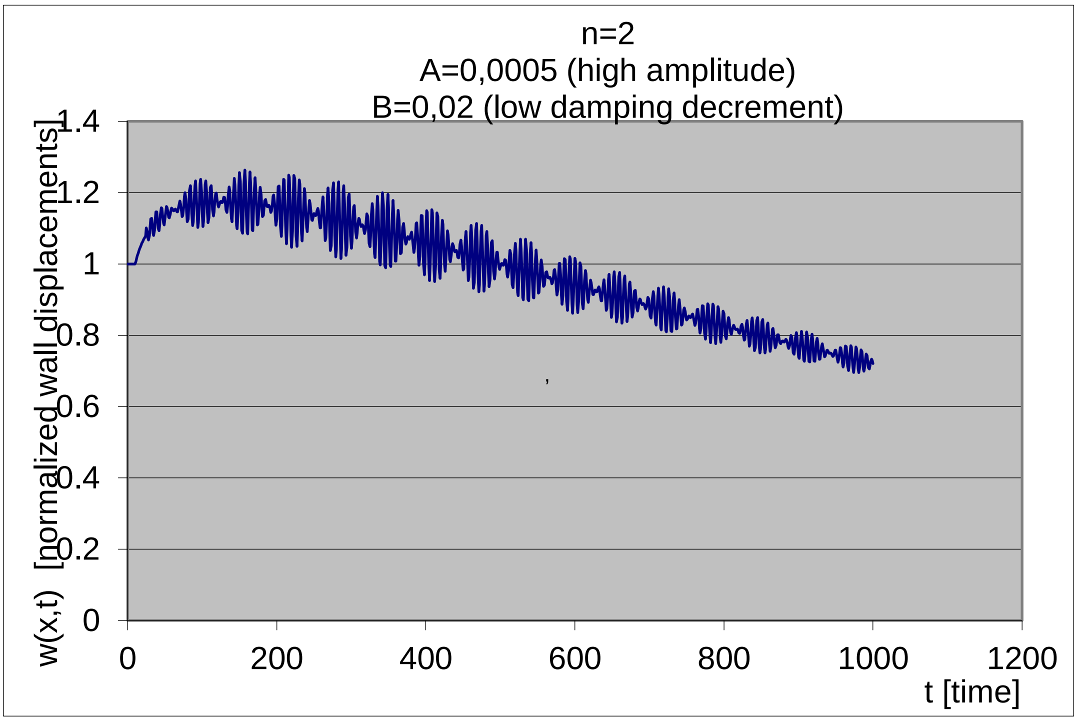

Fig. 11. Flexible silo wall with low damping decrement

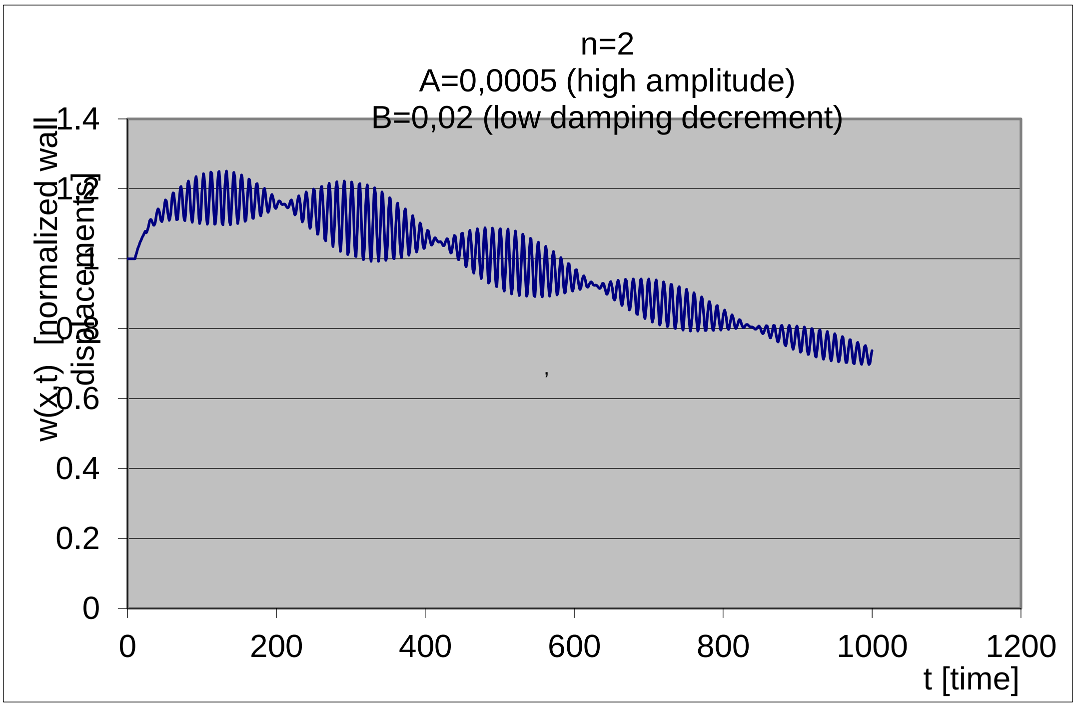

Fig.12. Poorly deformable wall with low damping decrement

The above examples, based on the available literature and research, seem to represent the most typical cases in the analysis of silo wall vibrations and pressures.

7. CONCLUSION

From the silo design point of view the problem of the pressures exerted on the walls of a silo is integrally bound with wall displacements and with strength and fatigue problems. The proposed model is to serve as a tool aiding the description and prediction of loads and some processes connected with the operation of reinforced concrete silos, steel silos and silos made of composite materials. The proposed way of modelling the strains and pressures was formulated as a component of the knowledge on the behaviour of the silo in the course of its operation. This particularly applies to the problem of the pressures exerted by the material on the silo walls. The problem receives much attention in specialist publications and seems to be crucial for the safety of silos. The model proposes a description which brings computational schemes closer to the actual performance of the silo.

In this paper, an attempt was made to reconcile the model's generality with its practical usefulness and so the model was limited to the analysis of the variation of only some selected parameters. But the model can be the starting point for a more detailed analysis of more complex cases or other time-variable processes which have not been considered in this paper.

The practical applications of the model are:

- the approximate description of vibration cycles with regard to vibration amplitudes and frequencies, which is needed when determining the fatigue strength of the silo's structural material (reinforced concrete, steel, a composite),
- the determination of the expected overloads under the action of the material on the silo wall.

The proposed model, based on the results of tests carried out on full-scale silos, still needs to be verified using a larger number of experimental descriptions for full-scale silos. It can also be the basis for formulating the way of carrying outrelevant tests and their range.

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